Cost-Benefit Analysis of Gas Hydrate Exploitation

Lena Döpke

CAU Kiel - Department of Economics

06.10.2008

Outline

Outline

Cake-Eating-Problem

Simplest of all non-renewable resource models:

$$\max_{q_t} W = \int_0^\infty U(q_t) \cdot e^{-\rho t} \mathrm{d}t$$
$$s.t. \quad \dot{x}_t = -q_t$$

x_t: stock of the resource at time t.
q_t: extraction rate at time t.
$U(q_t)$: utility function $(0 < U'(q) \leq \infty,\ U''(q) \leq 0)$
ρ: time preference rate / discount rate

Pollution Externalities

Simple non-renewable resource problem including pollution externalities:

$$
\begin{aligned}
\max_q W &= \int_0^\infty (U(q_t) - D(z_t)) \cdot e^{-\rho t} \mathrm{d}t \\
s.t. \quad \dot{x}_t &= -q_t \\
\dot{z}_t &= q_t - \alpha z_t
\end{aligned}
$$

z_t: stock of pollution in the atmosphere

α: rate of natural decay

$D(z_t)$: damage cost of pollution ($D'(z_t) > 0$, $D''(z_t) > 0$)

Outline

Outline

Pollution Externalities and Stock Effects

Modeling the risk of a tsunami via a stock dependent damage:

$$\begin{aligned} \max_q W &= \int_0^\infty (U(q_t) - D(x_t, z_t)) \cdot e^{-\rho t} \mathrm{d}t \\ s.t. \quad \dot{x}_t &= -q_t \\ \dot{z}_t &= q_t - \alpha z_t \end{aligned}$$

where $D_x(x,z) < 0,\ D_{xx}(x,z) \geq 0$

Current-value-Hamiltonian:

$$H(x_t, z_t, q_t, \lambda_t, \mu_t) = U(q_t) - D(x_t, z_t) - \lambda_t \underbrace{q_t}_{\dot{x}} - \mu_t \underbrace{(q_t - \alpha z_t)}_{\dot{z}}$$

$\Uparrow \qquad \Uparrow$

co-state variable λ_t: shadow value, opportunity cost
co-state variable μ_t: shadow cost $\hat{=}$ emission tax

Pollution Externalities and Stock Effects

Modeling the risk of a tsunami via a stock dependent damage:

$$\begin{aligned}\max_q W &= \int_0^\infty (U(q_t) - D(x_t, z_t)) \cdot e^{-\rho t} \mathrm{d}t \\ s.t. \quad \dot{x}_t &= -q_t \\ \dot{z}_t &= q_t - \alpha z_t\end{aligned}$$

where $D_x(x, z) < 0,\ D_{xx}(x, z) \geq 0$

Current-value-Hamiltonian:

$$H(x_t, z_t, q_t, \lambda_t, \mu_t) = U(q_t) - D(x_t, z_t) - \lambda_t \underbrace{q_t}_{\dot{x}} - \mu_t \underbrace{(q_t - \alpha z_t)}_{\dot{z}}$$

$\Uparrow$ $\Uparrow$

co-state variable λ_t: shadow value, opportunity cost

co-state variable μ_t: shadow cost $\hat{=}$ emission tax

Optimality Conditions

Necessary conditions for an interior optimum ($q(t) \geq 0$), if it exists, include:

$$\begin{aligned}
U'(q) &= \lambda + \mu && (1)\\
\dot{\lambda} &= \rho\lambda + D_x(x,z) && (2)\\
\dot{\mu} &= (\alpha+\rho)\mu - D_z(x,z) && (3)\\
\dot{x} &= -q && (4)\\
\dot{z} &= q - \alpha z && (5)\\
\text{and} \quad & \lim_{t\to\infty} x_t \lambda_t e^{-\rho t} = 0, \quad \lim_{t\to\infty} \mu_t e^{-\rho t} = 0 && (6)
\end{aligned}$$

Interpretations of Optimality Conditions

Along an optimal solution...

- ...the emission tax (shadow cost μ) is the discounted stream of marginal damage that a unit of emission today spills over to all future periods.
- ...the rise of the shadow value λ of the resource stock („scarcity rent") is damped by the stock-dependent damage.
- ...marginal benefit/utility (competitive price) always equals marginal social costs ($U'(q) = \lambda + \mu$).
- ...in general, depletion of the whole stock is not optimal.
- ...$U'(0) = -\frac{D_x(x^*)}{\rho}$ determines the stock x^* that is preserved forever.

Interpretations of Optimality Conditions

Along an optimal solution...

- ...the emission tax (shadow cost μ) is the discounted stream of marginal damage that a unit of emission today spills over to all future periods.
- ...the rise of the shadow value λ of the resource stock („scarcity rent“) is damped by the stock-dependent damage.
- ...marginal benefit/utility (competitive price) always equals marginal social costs ($U'(q) = \lambda + \mu$).
- ...in general, depletion of the whole stock is not optimal.
- ...$U'(0) = -\frac{D_x(x^*)}{\rho}$ determines the stock x^* that is preserved forever.

Interpretations of Optimality Conditions

Along an optimal solution...

- ...the emission tax (shadow cost μ) is the discounted stream of marginal damage that a unit of emission today spills over to all future periods.
- ...the rise of the shadow value λ of the resource stock („scarcity rent") is damped by the stock-dependent damage.
- ...marginal benefit/utility (competitive price) always equals marginal social costs ($U'(q) = \lambda + \mu$).
- ...in general, depletion of the whole stock is not optimal.
- ...$U'(0) = -\frac{D_x(x^*)}{\rho}$ determines the stock x^* that is preserved forever.

Interpretations of Optimality Conditions

Along an optimal solution...

- ...the emission tax (shadow cost μ) is the discounted stream of marginal damage that a unit of emission today spills over to all future periods.
- ...the rise of the shadow value λ of the resource stock („scarcity rent“) is damped by the stock-dependent damage.
- ...marginal benefit/utility (competitive price) always equals marginal social costs ($U'(q) = \lambda + \mu$).
- ...in general, depletion of the whole stock is not optimal.
- ...$U'(0) = -\frac{D_x(x^*)}{\rho}$ determines the stock x^* that is preserved forever.

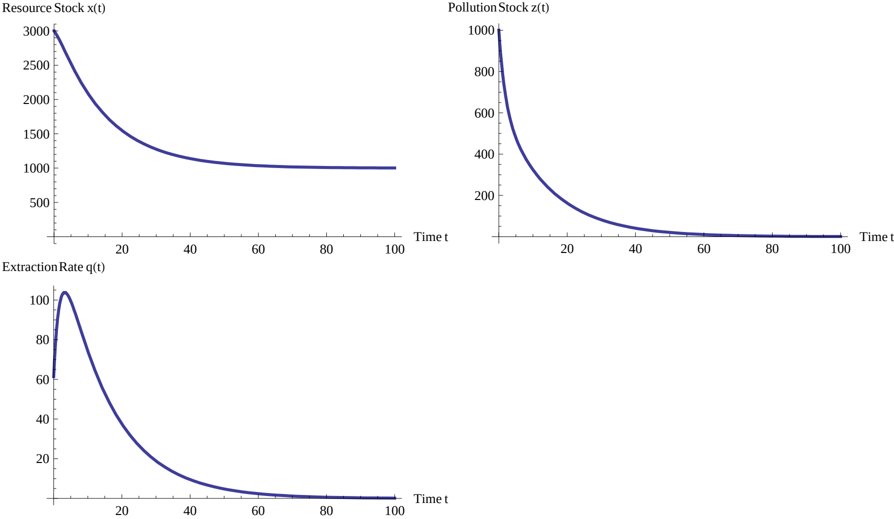
Interpretations of Optimality Conditions

Along an optimal solution...

- ...the emission tax (shadow cost μ) is the discounted stream of marginal damage that a unit of emission today spills over to all future periods.
- ...the rise of the shadow value λ of the resource stock („scarcity rent“) is damped by the stock-dependent damage.
- ...marginal benefit/utility (competitive price) always equals marginal social costs ($U'(q) = \lambda + \mu$).
- ...in general, depletion of the whole stock is not optimal.
- ...$U'(0) = -\frac{D_x(x^*)}{\rho}$ determines the stock x^* that is preserved forever.

Optimal Time Paths

pollution initially high:



Outline

Introduction

Non-Renewable Resource Model with Gas Hydrates

Tsunami-Risk

Global Warming Potentials

Blow-Outs

Conclusion

GWP + Extraction-Efficiency

$$\begin{aligned}
\max_q W &= \int_0^\infty \left(U(\varphi\, q_t) - D(x_t, z_t) \right) \cdot e^{-\rho t} \mathrm{d}t \\
s.t. \quad \dot{x}_t &= -q_t \\
\dot{z}_t &= \left(\varphi\kappa + (1-\varphi) \right) q_t - \alpha z_t
\end{aligned}$$

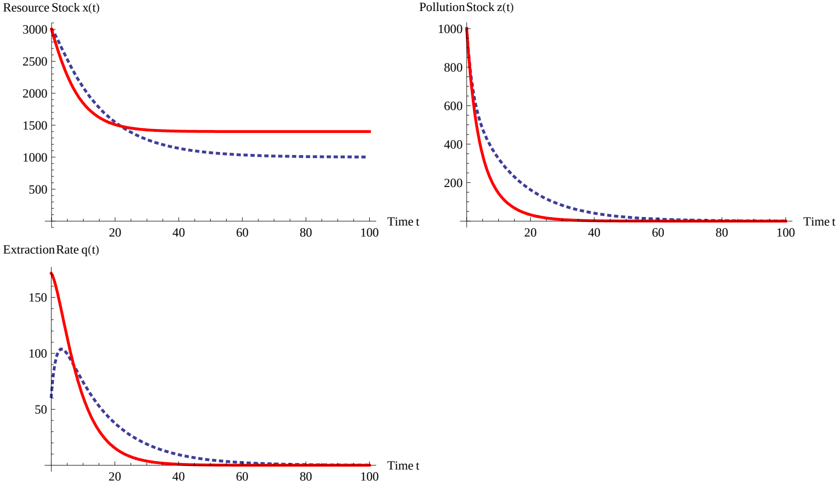
φ: extraction efficiency

$(1-\varphi)$: loss in the process of extraction

κ: coefficient to methane units

Results

pollution initially high:



Outline

Possible Modeling of Blow-Outs

$$\begin{aligned} \max_q W &= \int_0^\infty \left(U(\varphi\, q_t) - D(x_t, z_t)\right) \cdot e^{-\rho t} \mathrm{d}t \\ s.t. \quad \dot{x}_t &= -q_t - \beta x_t \\ \dot{z}_t &= \left((1 - \varphi(1-\kappa)\right) q_t - \alpha z_t + \beta x_t \end{aligned}$$

φ: extraction efficiency
$(1-\varphi)$: loss in the process of extraction
κ: coefficient to methane units
β: average rate of „natural“ decay.

Outline

- Thank you for your attention!